MODELING COUNSELING CENTER WAIT TIMES

by

Eloisa A. Sanchez

Honors Thesis

Appalachian State University

Submitted to the Department of Mathematical Sciences
and The Honors College
in partial fulfillment of the requirements for the degree of

Bachelor of Science

August, 2021

Approved by:

René Salinas, Ph.D., Thesis Director

Jacob Cooper, Ph.D., Second Reader

William J. Cook, Ph.D., Honors Director, Department of Mathematical Sciences

Jefford Vahlbusch, Ph.D., Dean, The Honors College

Abstract

The demand for counseling in colleges continues to increase, including Appalachian State University. Using data provided by Appalachian's counseling center, I create three simulations: the Individual Referral Simulation (IRS), the Top Heavy IRS, the Bottom Heavy IRS, and the Part-Time IRS. I found that increasing staff to hold only ten appointments more a week for individual counseling reduced wait times and allowed all referred students in the simulation to receive at least one appointment. If increasing staff is not possible, the Bottom Heavy IRS also results in a decrease of wait times, although not as much as the Part-Time IRS.

Contents

1 Introduction **1**

2 Problem **3**

3 Assumptions **4**

3.1 Data Provided by Counseling Center 4

3.2 Assumptions Based on Data 6

4 Methods **7**

5 Model **8**

5.1 Queuing Model for Initial Appointments 8

5.2 Simulating Appointment Availability 10

5.3 Model Modifications 14

5.3.1 Top Heavy IRS 14

5.3.2 Bottom Heavy IRS 17

5.3.3 Part-Time IRS 17

6 Results **17**

6.1 IRS Results 17

6.2 Top Heavy IRS Results 20

6.3 Bottom Heavy IRS Results 22

6.4 Part-Time IRS Results 22

7 Discussion **25**

8 Future Directions **27**

1 Introduction

Every year in the United States, one in five adults and one in six people between the ages of six and seventeen experience mental illness [5]. Of those, 50% of chronic mental illness establishes itself by the age of fourteen and 75% of cases establish themselves by the age of 24 [5]. This means that mental illness in a majority of Americans has presented itself by the typical age that many students start their college career [5]. Many colleges have identified the importance of mental health counseling, and nowadays students are able to access mental health resources through their college. Easy access is vital for college students to get mental health support through their academic career so their focus can be on their education. In fact, studies have found that students who take advantage of their college's mental health counseling show an improvement in academic performance [1]. Additionally, 49% of college students blamed their mental health as the main factor preventing them from meeting a higher level of academic performance [4]. In the California Community College System, an organization called the Mental Health and Wellness Association has been formed to focus on improving students' mental health, with student academic success and retention increasing as a result [4]. As such, accessibility to mental health counseling through college is vital to supporting students academically.

There has been an increase in demand by college students for mental health counseling in the past five years which cannot fully be explained by an increase in enrollment at universities, although it does play a role [7]. Additionally, students' mental health needs have increasing in seriousness and complexity [4]. Between 1988 and 2001, cases of anxiety in college students rose by almost 60%, depression by more than 50%, and mental disorders more than 50%, along with increases in chronic mental health problems, suicidal tendencies, and substance abuse [4]. Many colleges lack the resources necessary to increase their counseling center services at the rate that demand has risen, leading to staff shortages being reported by 88% of colleges [7]. As a result, these counseling centers have become overwhelmed and have long wait list times [7] [4]. In times of high demand, counseling centers are faced with the problem of optimizing

how many students can be seen to attempt and reduce the wait list times. As Appalachian State University increases the total student body to above 20,000, the demand for counseling will only continue to increase.

Many other health services have faced similar problems, attempting to reduce wait times by optimizing how patients are seen. Some health care settings have turned to creating models using queuing theory in order to reduce wait-times without increasing their staff. At the Lehigh Valley Health Network emergency department, a queuing model was developed to improve staffing according to patient-arrival demand with the added goal of not increasing resource hours overall [6]. They focused on three factors and their variations to then create correlations: patient arrival rate, server rate, and the number of servers available, servers being the staff attending to patients [6]. The first correlation is based on the emergency department's staffing plan and the average number of patients that arrive per hour to estimate the average server use [6]. The next correlation focuses on server use and the coefficients of variation of arrival and server rates to estimate the number of patients in the queue [6]. Lastly, the third correlation is based on a theorem that states the average number of patients at a location is equal to the arrival rate times the average time the patient is receiving services, Little's Law, which helps estimate the average wait times at different times of day [6]. After collecting data on the emergency department, they found that there were not enough providers and RNs on staff between 7 am and 2 pm, with resource usage being close to 100 percent or above [6]. After adjusting the staffing schedule based on the demand at different hours without increasing staffing hours, the model predicts that the highest wait times would be around 30 minutes, and less than three patients would waiting at a time [6]. For Lehigh Valley Network's emergency department, using a queuing model was effective at reducing wait times simply by manipulating staffing without hiring new staff.

However, a combination of queuing theory and simulations proves most effective for making suggestions on how to reduce wait times in some settings. In Iran, the Shahid Faghihi Hospital Emergency Department is experiencing an overcrowding crisis caused by a combina-

tion of high rates of arrivals and a shortage of inpatient beds [3]. No patients can be turned away from the emergency department and all patients must be evaluated and referred or discharged within six hours [3]. Due to the bed shortage, patients in the emergency department must stay longer than would be expected otherwise [3]. After collecting the data from the hospital's records and through sampling random days in the emergency department, the information was input into a simulation software called Arena [3]. The model was run for a month and after testing several scenarios, the results demonstrated that techniques to reduce the total length of stay would help reduce the queue but the most effective action would be to more than double the amount of beds in the emergency department [3]. However, increasing beds would also require funding to not only buy more beds but also increase staff in the emergency department to attend to the beds. Although the researchers were able to consider other options that would not require allocating a large amount of funding, those scenarios did not address the root of the problem, which is a lack of beds [3]. However, the solution for the Shahid Faghihi Hospital emergency department requires an increase in funding, beds, and staff, unlike the Lehigh Valley Network's emergency department [3]. By not relying solely on queuing theory and involving a simulation, the researchers were able to make several suggestions on how to reduce the queue, even though the most effective solution would be to buy more beds [3]. As such, although queuing models can help locate where a lack of resources exists, involving simulations can also help test different scenarios that could potentially reduce queues.

2 Problem

In order to consider ways to optimize staffing to reduce wait times in a counseling center, I will model the process a student goes through to receive individual counseling and the wait times associated with the process. At Appalachian, students must set up an initial evaluation to be referred to group therapy, individual therapy, or other alternatives for seeking help, such as an off-campus resource that better suits their needs. If a student is referred to individual

counseling, they must wait to be assigned to a counselor for their first appointment. After their first appointment, they can attend up to ten appointments. Once a student is assigned an appointment slot, they typically keep the same slot until they decide to stop receiving counseling or reach ten appointments. When a student ends their time at the counseling center, they can choose to be referred to other resources, whether on-campus or off-campus, but that is not required. It is important to note that at Appalachian the ten appointment limit was lifted for the 2020-2021 academic year due to the pandemic but it is an exception to the typical academic year, so any data for ten or more appointments will be combined to better represent a typical academic year.

The problem of interest is the wait that occurs between when a student receives a referral for individual counseling and when a student attends their first appointment. While the initial evaluation is typically conducted almost immediately after a student requests it or the next available time that works for the student, the wait for the first individual counseling appointment depends upon both how long it takes for an appointment slot to open up and the number of students that are already waiting for their first appointment. Once all appointment slots are filled and a wait list begins, the wait time has the potential to build upon itself and result in students who have to wait weeks for their first appointment. My goal is to create a model and analyze different scenarios that could improve wait times for students.

3 Assumptions

3.1 Data Provided by Counseling Center

In order to gather data, I reached out to the director of the counseling center at Appalachian, Christopher Hogan. Through both written and spoken correspondence, I was able to learn the following about individual appointments between August 17, 2020 and April 28, 2021:

- A total of 4,821 individual therapy appointments were attended

- 990 unique students attended individual therapy appointments.
- The average number of individual therapy appointments a student attends is 4.87.
- The average number of days between the initial evaluation and scheduling the first individual therapy appointment is 4.78 days
- Students are mainly scheduled on a bi-weekly basis (every two weeks.)
- Traditionally, the demand for individual therapy appointments is high the first few weeks of the semester, stabilizing still high but a little lower than the first few weeks, and then reducing the last few weeks of each semester.
  - Specifically for the given time frame, each semester began with high demand, peaking the most the fifth and sixth week of the fall semester and the fourth and fifth weeks of the spring semester. For both semesters, the demand remains high with another spike around the eighth and ninth weeks of both semesters, then reducing the last weeks of the semester.
- A portion of students during the winter break decide to not return the spring semester.
- Director Hogan also provided the percentage of students that stop after n appointments, as summarized in Figure 1 below:

1	2	3	4	5	6	7	8	9	10	11	12+
12.6%	17.7%	14%	11.9%	11.1%	6.7%	5.6%	3.9%	4.4%	5.6%	2.2%	4.4%

Figure 1: Percentages of Students who stop after n Appointments

In order to model the problem of interest, as with any other model, I will make assumptions about the problem. Many of the following assumptions will involve the information provided by Director Hogan in order to reflect a counseling center run similarly to Appalachian's.

3.2 Assumptions Based on Data

1. The referral process to individual counseling is quick and done either immediately or soon after a request for an initial assessment and does not have a strong impact on wait-times. This assumption serves to simplify the model since the initial evaluation does not have an impact on the wait time that occurs after the referral.

2. Appointments are bi-weekly, with sixteen weeks per semester. Although not all appointments are scheduled bi-weekly at Appalachian, since most students attend appointments on a bi-weekly basis, this assumption will serve to simplify the model.

3. Students can only attend up to ten sessions. This past academic year, the ten session limit at Appalachian was waived due to the pandemic. To allow the model to apply to how the counseling center is typically run, all data on ten or more sessions will be combined.

4. There is the equivalent of 9 full-time workers, each available for 20 appointments a week. Since there are both full-time and part-time workers in counseling centers, assuming a certain number of full-time equivalent workers simplifies the number of starting available number of appointments.

5. Since the actual percentage of students that do not return for services after the break between semesters is not known, I will assume a 20% loss to account for students who found counseling services off-campus and students who did not return for the second semester.

6. The percent of students who stop after n appointments does not vary from year to year. Although the data provided by Director Hogan is focused on the past academic year, this assumption serves to qualify the data as representative of a typical year at the counseling center.

7. Students do not leave the wait list other than during the break. Assuming students only leave the list during the break serves to simplify the model.

8. All students attend their scheduled sessions. This is also a simplifying assumption.

4 Methods

At first glance, in order to model the entire counseling center process and its wait times, it appeared that queuing theory would apply well to the problem since after receiving a referral students wait to be assigned their first appointment, which is where the queue begins. However long it would take for students to be assigned to their first appointment would depend upon the number of counselors and how long it takes for appointments to become available. Once a student is assigned their first appointment, they have the option to attend up to ten appointments, which is where the idea of using a classic queuing model fails.

Although a queuing model can account for multiple phases of service before a client leaves the system, I found no previously established queuing model nor research that considers a scenario where the wait time in a queue depends on a process that waits for appointments to open up from a client leaving after attending several appointments. Thus, it became clear that the problem would need to extend beyond queuing theory alone. In order to model a student waiting to be assigned to a counselor and then attending their first appointment with a queuing model, there would have to be extensive research conducted to consider how repeated appointments could be modeled using queuing theory. Despite the process of a student arriving to the counseling center, setting up an initial evaluation, and then being referred for individual therapy can be modeled by a simple queuing model, that is not the problem of interest. Instead, I will briefly go over how a queuing model can be applied to the first steps before a student is assigned their first counseling appointment and then create a simulation based off data given to me about Appalachian's counseling center by Director Hogan.

5 Model

5.1 Queuing Model for Initial Appointments

Simple queuing models are based on data about distribution of interarrival times, distribution of service times, the amount of servers, the queuing discipline, and the maximum population accepted. Interarrival time is the time between arrivals, often modeled with an exponential distribution since it can model the likelihood of another arrival over time after an arrival without depending on how much time has passed since the last arrival. The interarrival time is often also modeled by a Poisson distribution that calculates the probability of a certain number of arrivals within a given time period. In cases where an exponential distribution does not apply because arrivals are not random, an Erlang distribution or general distribution can be used in situations where arrivals are scheduled or regulated but are not useful for the purposes of my work [2]. The same distributions are often used for the service times when the service times are not constant and can vary per client. The amount of servers is based on the scenario the queuing model is being applied to. The queuing discipline has to do with the order in which clients are attended to, such as first come, first serve (FCFS), last come, first served (LCFS), and service in random order (SIRO). As for the maximum population that could enter the queue, that also depends on the scenario to which the queuing model is being applied to. Rather than having to write that all out to describe a queuing model, notation has been created to summarize the details of a queuing model. The order of the notation goes interarrival distribution / service time distribution / number of servers / queuing discipline / customer pool size. For the distributions, M refers to an exponential distribution (including the use of Poisson distribution), E is for Erlang distribution, and G stands for general distribution. Some of the most common queuing disciplines are FCFS, LCFS, and SIRO which are represented with their respective acronyms. Both the number of servers and customer pool size rely on the problem, although the customer pool size is often simplified to be ∞.

Arguably the most basic queuing model is an M/M/1/FCFS/∞ model. An example of an

M/M/1 setting is a singular vending machine serving customers. The one server would be the vending machine. Since the interarrival times and service times are not regulated, an exponential distribution is the most appropriate distribution. Additionally, customers are served FCFS and as many people that want to can wait in a queue to use the vending machine, so the calling population would be ∞. Although this example can be considered a simple situation, possible areas of improvement could be increasing the number of vending machines or improving the service method so customers spend less time at the vending machine. In models where there is more than one server, the problems tend to be more complex.

A common queuing model is the M/M/s/FCFS/∞ model, which is commonly referred to as a M/M/s model when the queuing discipline and calling population are FCFS and ∞ respectively. For example, in 2013, the KeyCorp bank holding company turned to an Operational Research study with the goal that 90% of customers would have to wait less than five minutes for service while also maintaining cost-effective staffing [2]. To analyze the problem, a M/M/s queuing model was applied. In order to apply the model, data was collected about the average service time, 246 seconds, and mean arrival rates [2]. With that data, the model returned that KeyCorp would have to increase the amount of tellers by 30% to meet their goal [2]. However, this result failed to maintain cost-effective staffing, so KeyCorp instead worked to change the structure of their customer sessions and their staff management over the span of three years [2]. As a result of their re-structuring, the average service time was reduced to 115 seconds [2]. Re-applying the M/M/s queuing model, it consistently returned that their service time goal could be surpassed while also reducing staff by improving scheduling among various bank branches [2]. Consequently, KeyCorp saved almost $20 million per year and 96% of their customers wait less than five minutes [2].

As discussed in the problem statement, when students first arrive to the counseling center at Appalachian State University, they must attend an initial assessment session that is used to determine whether a student should be referred to group therapy, individual therapy, or to another resource. Afterwards, they have to wait to be assigned their first appointment with a counselor.

A queuing model fits well for the initial assessment process, with interarrival times that fit the traditional Poisson distribution, service time is approximately constant (each initial assessment is 50 minutes long), students are serviced FCFS, and the calling population is "infinite." While I am not sure exactly how many people on staff conduct initial assessments at a time, with that number, I would know the number of servers, and I would have a complete queuing model. For the initial intake in the counseling center, the notation would be M/M/s/FCFS/∞. Since there are cyclic-like changes in demand throughout the year, if this were the problem of interest, I could attempt a distribution where the mean changes over time in a cyclic fashion since the traditional Poisson distribution has a constant mean.

As explained earlier, my initial goal was to create a queuing model that fits the next step of the process: waiting to be assigned the first appointment with a counselor and then attending up to ten sessions. However, I found no examples of a classic queuing model that has the facility to calculate approximate wait-times by factoring in repeated appointments. Since this part of the process is the problem of interest, I decided to create a simulation instead.

5.2 Simulating Appointment Availability

For the second, most interesting part of my model, I reached out to Director Hogan of Appalachian's counseling center, and I learned the percentages of students who stop after n number of appointments, summarized in Figure 2 (typically students are only allowed to have up to ten appointments, but the appointment limit was waived due to the pandemic. To comply with my third assumption, I combined the data on ten or more appointments.) Within the given time-frame, there were 990 unique students who attended individual therapy. Demand was high at the start of the semester with the greatest demand being between five and six weeks into the fall semester and between four and five weeks for the spring semester. The demand decreases slightly but remains high with another spike around the eighth or ninth week of both semesters, and reducing around the last two weeks of the semester. During spoken correspondence, Director Hogan also said that a notable portion of students do not continue their sessions

1	2	3	4	5	6	7	8	9	10+
12.6%	17.7%	14%	11.9%	11.1%	6.7%	5.6%	3.9%	4.4%	12.2%

Figure 2: Modified Percentages of Students who stop after n Appointments

after winter break and that he schedules mainly on a bi-weekly basis.

To create a simulation modeled after the counseling center at Appalachian, called the Individual Referral Simulation (IRS), I created a spreadsheet. The first column in Figure 3, Week, keeps track of how many weeks into a semester the simulation is. Each row covers the span of two weeks since Director Hogan said that most students are scheduled on a bi-weekly basis, so if their first appointment occurs during the first week of the semester, their next appointment would be the third week of the semester. By grouping weeks in pairs, I prevent complications from having to alternate between students who have appointments on odd-numbered weeks and those who have appointments on even-numbered weeks. The number of weeks per semester reflects the sixteen weeks Appalachian has per semester. Additionally, there is an entry called Break which represents winter break, not a two-week span.

The second column, students referred for individual appointments, follows the demand pattern described by Director Hogan. The numbers themselves are arbitrary, although chosen to also result in around 990 students served since that is the number of students that were served the past academic year. Following the pattern Director Hogan provided, the highest number of students referred for the first semester occurs during Week 5, the second peak being slight lower during the 9th week. The second semester, Week 3 has the first peak, with a slightly lower peak Week 9. Both semesters have a decline after the second peak.

The third column, referrals accommodated, is the number of students for the respective week set that were able to attend their first appointment. Especially once there are not enough available appointments at the start of a week set, this column will allow one to compare the difference between the students referred and the students that were accommodated. When the sum of students waiting and students referred is less than the available appointments, the

referrals accommodated is the sum of students waiting and students referred. When the amount of available appointments is less than the sum, then the amount of available appointments is the number of referrals accommodated, with priority given to the students that have been waiting the longest.

The waiting column keeps a cumulative count of the students that have not yet been accommodated from previous weeks and the new students that have entered the wait list that week set. Once students from the wait list are accommodated, they no longer are included in the count.

The following ten columns, named first, second, third, ..., tenth, calculate the number of students that are attending their first, second, third, ..., tenth appointments. The number of students attending their first appointment depends on the number of available appointments at the start of the week set. If the sum of students waiting and students referred is less than the number of available appointments, then the sum is the number of students that attend their first appointment. If the sum is greater than the number of available appointments, then the number of available appointments is how many students who received their first appointment that week set. To calculate the number of students that continue onto their n^{th} appointment, the $n - 1$ appointment's number of students from the previous week set is multiplied by 1 - the respective percentage of students who stop after n appointments from Figure 2. For instance, if 40 students attended their first appointment the previous week set, to calculate the number of students that attend their second appointment, I would use the following formula: $40 * (1 - .177)$.

The last column, number of vacant appointments, calculates the number of available appointments at the end of the week set the row corresponds to. As such, it is also the number of available appointments at the start of the following week set. By taking the sum of the "first" through "tenth" columns and subtracting it from 360, the total number of appointments per week set, I found the number of vacant appointments at the end of week set.

To initialize, I began with no referrals and no appointment slots taken. For the simulation,

			Number of appointments											
Week	students referred for individual appointments	referrals accomodated	waiting	first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth	number of vacant appointments
initialize	0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	360.00
1	70	70.0	0.00	70.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	290.00
3	80	80.0	0.00	80.00	61.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	218.82
5	100	100.0	0.00	100.00	69.92	50.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	139.61
7	80	80.0	0.00	80.00	87.40	57.68	43.41	0.00	0.00	0.00	0.00	0.00	0.00	91.51
9	90	90.0	0.00	90.00	69.92	72.11	49.61	38.24	0.00	0.00	0.00	0.00	0.00	40.13
11	60	40.1	19.87	40.13	78.66	57.68	62.01	43.70	34.00	0.00	0.00	0.00	0.00	43.82
13	50	43.8	26.06	43.82	35.07	64.89	49.61	54.63	38.85	31.72	0.00	0.00	0.00	41.41
15	20	20.0	4.65	41.41	38.30	28.93	55.81	43.70	48.57	36.25	29.94	0.00	0.00	37.09
Break			3.72	33.12	30.64	23.15	44.65	34.96	38.85	29.00	23.95	0.00	0.00	101.67
1	60	60.0	0.00	63.72	28.95	25.28	19.91	39.33	31.08	36.25	27.38	23.02	0.00	65.08
3	90	65.1	24.92	65.08	55.69	23.88	21.74	17.54	34.97	29.00	34.22	26.31	20.19	31.38
5	70	31.4	63.54	31.38	56.88	45.95	20.54	19.15	15.59	32.63	27.38	32.89	23.07	54.55
7	70	54.5	78.99	54.55	27.43	46.93	39.51	18.10	17.03	14.55	30.80	26.31	28.84	55.97
9	80	56.0	103.02	55.97	47.68	22.63	40.36	34.81	16.09	15.88	13.73	29.60	23.07	60.19
11	60	60.0	102.83	60.19	48.92	39.33	19.46	35.56	30.95	15.01	15.00	13.20	25.96	56.45
13	20	20.0	66.39	56.45	52.60	40.36	33.83	17.14	31.61	28.87	14.17	14.41	11.57	58.99
15	10	10.0	17.40	58.99	49.33	43.40	34.71	29.80	15.24	29.49	27.26	13.62	12.64	45.53
	Total referred:	Total Served:												
	1010.0	991.7												

Figure 3: Individual Referral Simulation

complying with assumption 4, there are nine full-time employees, each able to hold twenty appointment slots every week, resulting in 360 appointments for every week set. Since a portion of students do not tend to return after the break between semesters, the row that represents the break takes the numbers from the last week of the semester and multiplies each by .8, representing a 20% loss of students waiting for and attending appointments, as established in assumption 5. The resulting numbers initialize the start of the second semester. The calculations for the IRS are summarized by the flow diagram in Figure 4.

5.3 Model Modifications

To compare alternative forms of staffing, I created three additional simulations. Two of the simulations do not increase staffing, and one considers increasing the available appointments by increasing staff slightly. For each modified simulation, the set-up is the same as the IRS, the difference mainly relying on the number of available appointments.

5.3.1 Top Heavy IRS

For the first modification, assumption 4 is changed. Rather than there being the equivalence of nine full-time workers for the academic year, I will assume that there are ten full-time workers the first semester and eight the second semester. By shifting a counselor from one semester to the other, the total number of staff is left unchanged but impacts the amount of available appointments for each semester. The academic year would begin with 400 appointments available and after the break there would be 320 appointments available. Ideally, focusing more resources the first semester would prevent a build-up of wait times later in the year. After running the simulation, the Top Heavy IRS, it results in 930.9 students being served, less that the IRS, seen in Figure 5.

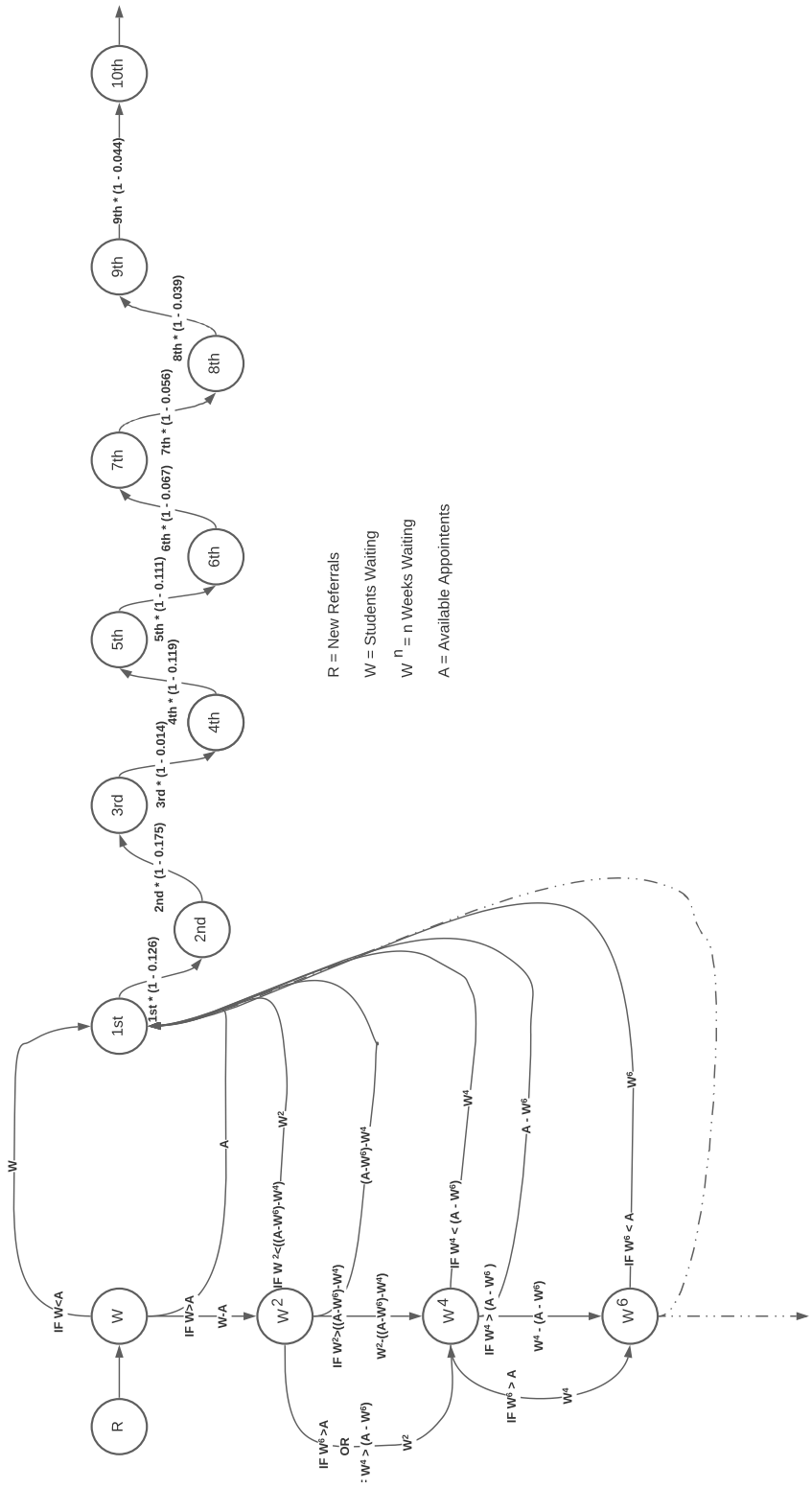

Figure 4: IRS Flow Diagram

		Number of appointments												
Week	students referred for individual appointments	new referrals accomodated	waiting	first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth	number of vacant appointments
initialize	0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	400.00
1	70	70.0	0.00	70.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	330.00
3	80	80.0	0.00	80.00	61.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	258.82
5	100	100.0	0.00	100.00	69.92	50.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	179.61
7	80	80.0	0.00	80.00	87.40	57.68	43.41	0.00	0.00	0.00	0.00	0.00	0.00	131.51
9	90	90.0	0.00	90.00	69.92	72.11	49.61	38.24	0.00	0.00	0.00	0.00	0.00	80.13
11	60	60.0	0.00	60.00	78.66	57.68	62.01	43.70	34.00	0.00	0.00	0.00	0.00	63.94
13	50	50.0	0.00	50.00	52.44	64.89	49.61	54.63	38.85	31.72	0.00	0.00	0.00	57.85
15	20	20.0	0.00	20.00	43.70	43.26	55.81	43.70	48.57	36.25	29.94	0.00	0.00	78.76
Break			0.00	16.00	34.96	34.61	44.65	34.96	38.85	29.00	23.95	0.00	0.00	63.01
1	60	60.0	0.00	60.00	13.98	28.84	29.76	39.33	31.08	36.25	27.38	23.02	0.00	30.34
3	90	30.3	59.66	30.34	52.44	11.54	24.80	26.22	34.97	29.00	34.22	26.31	20.19	29.96
5	70	30.0	99.69	29.96	26.52	43.26	9.92	21.85	23.31	32.63	27.38	32.89	23.07	49.20
7	70	49.2	120.49	49.20	26.19	21.88	37.21	8.74	19.43	21.75	30.80	26.31	28.84	49.65
9	80	49.7	150.83	49.65	43.00	21.61	18.82	32.78	7.77	18.13	20.53	29.60	23.07	55.04
11	60	55.0	155.79	55.04	43.40	35.48	18.58	16.58	29.14	7.25	17.11	19.73	25.96	51.73
13	20	20.0	124.06	51.73	48.11	35.80	30.51	16.37	14.74	27.19	6.84	16.44	17.30	54.96
15	10	10.0	79.10	54.96	45.22	39.69	30.79	26.88	14.55	13.75	25.67	6.58	14.42	47.50
	Total referred:	Total Served:												
	1010.0	930.9												

Figure 5: Top Heavy IRS

5.3.2 Bottom Heavy IRS

The second modification also changes assumption 4. This time, there are eight full-time equivalent workers the first semester and ten the second semester. Consequently, the first semester has 320 available appointments and the second has 400 available appointments. Although there was a shift in staffing, there was no increase in staff. Since the Top Heavy IRS resulted in less students who were served than the IRS even though the goal was to get ahead of growing wait times, the second modification, the Bottom Heavy IRS (Figure 6), tests shifting resources from the first semester to address the wait times that build up during the second semester. Running the simulation returns a higher number of students that were served the IRS, with a total of 998.9 students who received at least one appointment.

5.3.3 Part-Time IRS

The third simulation, the Part-Time IRS (Figure 7), considers adding a part-time counselor for the whole academic year that only holds ten appointments per week, so each semester there are 380 available appointments. As such, assumption 4 is changed once again, but unlike the other two modifications, the total staff would be increased. Although increasing staff would likely mean requiring an increase in budget, the goal is that an increase of even ten available appointments for the year will improve wait times for both semesters. Indeed, the simulation results in all students who were referred, 1,010 students, receiving at least one appointment, greater than both the IRS and the Bottom Heavy IRS.

6 Results

6.1 IRS Results

The IRS has a total of 1,010 students referred and 360 available appointments every two-week span. Of the students referred, 991.7 were able to attend at least one appointment. The last

		Number of appointments												
Week	students referred for individual appointments	new referrals accomodated	waiting	first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth	number of vacant appointments
initialize	0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	320.00
1	70	70.0	0.00	70.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	250.00
3	80	80.0	0.00	80.00	61.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	178.82
5	100	100.0	0.00	100.00	69.92	50.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	99.61
7	80	80.0	0.00	80.00	87.40	57.68	43.41	0.00	0.00	0.00	0.00	0.00	0.00	51.51
9	90	51.5	38.49	51.51	69.92	72.11	49.61	38.24	0.00	0.00	0.00	0.00	0.00	38.62
11	60	38.6	59.87	38.62	45.02	57.68	62.01	43.70	34.00	0.00	0.00	0.00	0.00	38.97
13	50	39.0	70.91	38.97	33.75	37.14	49.61	54.63	38.85	31.72	0.00	0.00	0.00	35.33
15	20	20.0	55.58	35.33	34.06	27.84	31.94	43.70	48.57	36.25	29.94	0.00	0.00	32.36
Break			44.46	28.26	27.25	22.28	25.55	34.96	38.85	29.00	23.95	0.00	0.00	169.89
1	60	60.0	0.00	104.46	24.70	22.48	19.16	22.51	31.08	36.25	27.38	23.02	0.00	88.96
3	90	89.0	1.04	88.96	91.30	20.38	19.33	16.88	20.01	29.00	34.22	26.31	20.19	33.42
5	70	33.4	37.62	33.42	77.75	75.32	17.53	17.03	15.00	18.67	27.38	32.89	23.07	61.94
7	70	61.9	45.68	61.94	29.21	64.14	64.78	15.44	15.14	14.00	17.63	26.31	28.84	62.57
9	80	62.6	63.11	62.57	54.13	24.10	55.16	57.07	13.73	14.13	13.21	16.94	23.07	65.88
11	60	60.0	57.23	65.88	54.69	44.66	20.73	48.60	50.73	12.81	13.34	12.70	14.86	61.01
13	20	20.0	16.21	61.01	57.58	45.12	38.41	18.26	43.20	47.34	12.09	12.82	11.14	53.04
15	10	10.0	0.00	26.21	53.33	47.50	38.80	33.84	16.23	40.31	44.68	11.62	11.24	76.24
	Total referred:	Total Served:												
	1010.0	998.9												

Figure 6: Bottom Heavy IRS

			Number of appointments											
Week	students referred for individual appointments	new referrals accomodated	waiting	first	second	third	fourth	fifth	sixth	seventh	eighth	ninth	tenth	number of vacant appointments
initialize	0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	380.00
1	70	70.0	0.00	70.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	310.00
3	80	80.0	0.00	80.00	61.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	238.82
5	100	100.0	0.00	100.00	69.92	50.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	159.61
7	80	80.0	0.00	80.00	87.40	57.68	43.41	0.00	0.00	0.00	0.00	0.00	0.00	111.51
9	90	90.0	0.00	90.00	69.92	72.11	49.61	38.24	0.00	0.00	0.00	0.00	0.00	60.13
11	60	60.0	0.00	60.00	78.66	57.68	62.01	43.70	34.00	0.00	0.00	0.00	0.00	43.94
13	50	43.9	6.06	43.94	52.44	64.89	49.61	54.63	38.85	31.72	0.00	0.00	0.00	43.91
15	20	20.0	0.00	26.06	38.41	43.26	55.81	43.70	48.57	36.25	29.94	0.00	0.00	58.00
Break			0.00	20.84	30.73	34.61	44.65	34.96	38.85	29.00	23.95	0.00	0.00	122.40
1	60	60.0	0.00	60.00	18.22	25.35	29.76	39.33	31.08	36.25	27.38	23.02	0.00	89.60
3	90	89.6	0.40	89.60	52.44	15.03	21.80	26.22	34.97	29.00	34.22	26.31	20.19	30.22
5	70	30.2	40.18	30.22	78.31	43.26	12.93	19.21	23.31	32.63	27.38	32.89	23.07	56.80
7	70	56.8	53.38	56.80	26.41	64.61	37.21	11.39	17.07	21.75	30.80	26.31	28.84	58.81
9	80	58.8	74.56	58.81	49.65	21.79	55.56	32.78	10.12	15.93	20.53	29.60	23.07	62.16
11	60	60.0	72.41	62.16	51.40	40.96	18.74	48.95	29.14	9.45	15.04	19.73	25.96	58.48
13	20	20.0	33.92	58.48	54.32	42.41	35.22	16.51	43.52	27.19	8.92	14.45	17.30	61.68
15	10	10.0	0.00	43.92	51.11	44.82	36.47	31.03	14.68	40.60	25.67	8.57	12.67	70.46
	Total referred:	Total Served:												
	1010.0	1010.0												

Figure 7: Part-Time IRS

two-week span of the second semester has 17.4 students still waiting for their first appointment. Realistically, unless the students decide to receive counseling during a summer semester, those students would be referred to outside resources. When students enter the wait list, they immediately wait for at least two weeks they must wait to see if there are enough available appointments for them to receive their first appointment the following week set. If there are students that have been waiting for a longer length of time the following week set, those students are prioritized. For instance, if there are thirty students who have been on the wait list for four weeks, forty students who have waited for two weeks, thirty new referrals, and fifty available appointments, all thirty students who have waited four weeks will receive their first appointment. Twenty of the forty students who have waited for two weeks will also receive their first appointment. The other twenty will remain on the wait list, and the thirty new referrals will join the wait list. The formulas used to determine who gets priority for first appointments is summarized in Figure 4.

In the simulation, a wait list did not start until the 11th week with 19.87 students, as seen in Figure 8. The 13th week, all 19.87 students attended their first appointment, as noted in Figure 8. Many of the following weeks have students who receive their first appointment after waiting two weeks, but the 11th week of the second semester there are 8.99 students who attend their first appointment that had to wait four weeks, along with 46.98 students who waited for two weeks before attending their first appointment. The rest of the weeks, there are students who have waited at least two weeks but at most four for their first appointment. Additionally, since there were 4.65 students waiting before the break, 20% did not return, meaning there were some students that left the simulation before their first appointment. At the end of the simulation, there were still 17.40 students on the wait list.

6.2 Top Heavy IRS Results

The Top Heavy IRS saw 930.9 students attended to, 60.8 students less than the original IRS, and had 400 available appointments the first semester and 320 the second semester. The wait times

Week	Two weeks waiting	Four weeks waiting	Six weeks waiting
0	0.00	0.00	0.00
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	0.00	0.00	0.00
7	0.00	0.00	0.00
9	0.00	0.00	0.00
11	0.00	0.00	0.00
13	19.87	0.00	0.00
15	26.06	0.00	0.00
Break	Break	Break	Break
1	3.72	0.00	0.00
3	0.00	0.00	0.00
5	24.92	0.00	0.00
7	54.55	0.00	0.00
9	46.98	8.99	0.00
11	37.17	23.02	0.00
13	13.61	42.83	0.00
15	12.60	46.39	0.00

Figure 8: Individual Referral Simulation Wait list

for this simulation were calculated the same way as the wait time for the IRS was. Although no students have to wait past two weeks for their first appointment in the first semester, the second semester has students who have to wait up to six weeks for their first appointment. By the 9th week, the wait list had built up enough that there were only enough available appointments for students who had waited four weeks, so no students who had waited two weeks were able to get their first appointment, as seen in Figure 9. By the end of the semester, no students past the 7th week who had waited for only two weeks were able to get their first appointment, waiting at minimum four weeks. The last two weeks of the second semester, 79.1 students are still waiting for an appointment. In this scenario, no students were on the wait list on the break, so no students left the simulation before receiving their first appointment.

6.3 Bottom Heavy IRS Results

On the other hand, the Bottom Heavy IRS saw 998.9 students served, 7.2 students more than the original IRS. This simulation had 320 appointments available the first semester and 400 the second. Although it is a small margin above the IRS, there is only one group of students that have to wait more than two weeks for their first appointment throughout the academic year, and the academic year ends with no students waiting for an appointment, as noted in Figure 10. However, since there was a wait before the break, some students never attended their first appointment, meaning that although all the students who remained on the wait list were served, some students were still lost because there was a wait before the break.

6.4 Part-Time IRS Results

Lastly, the Part-Time IRS attended to every student who was referred to individual counseling by having 380 available appointments for both semesters. Like the Bottom Heavy IRS, there is only one group of students that has to wait for more than two weeks for their first appointment and no students are left waiting. Additionally, looking at Figure **??** and comparing it with the

Week	Two weeks waiting	Four weeks waiting	Six weeks waiting
0	0.00	0.00	0.00
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	0.00	0.00	0.00
7	0.00	0.00	0.00
9	0.00	0.00	0.00
11	0.00	0.00	0.00
13	0.00	0.00	0.00
15	0.00	0.00	0.00
Break	Break	Break	Break
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	29.96	0.00	0.00
7	19.51	29.69	0.00
9	0.00	49.65	0.00
11	0.00	54.21	0.83
13	0.00	36.77	14.96
15	0.00	10.90	44.06

Figure 9: Top Heavy IRS Waitlist

Week	Two weeks waiting	Four weeks waiting	Six weeks waiting
0	0.00	0.00	0.00
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	0.00	0.00	0.00
7	0.00	0.00	0.00
9	0.00	0.00	0.00
11	38.49	0.00	0.00
13	38.97	0.00	0.00
15	14.42	20.91	0.00
Break	Break	Break	Break
1	44.46	0.00	0.00
3	0.00	0.00	0.00
5	1.04	0.00	0.00
7	37.62	0.00	0.00
9	45.68	0.00	0.00
11	63.11	0.00	0.00
13	57.23	0.00	0.00
15	16.21	0.00	0.00

Figure 10: Bottom Heavy IRS Waitlist

figures in Figure 10, the group of students that waited for four weeks was less in the Part-Time IRS. In the Bottom Heavy IRS, 20.91 students had to wait four weeks while in the Part-Time IRS, 12.41 students waited four weeks. As such, less students had to wait four weeks. Additionally, since there was no wait before the break no students left without having a first appointment, allowing all students to receive at least their first appointment.

7 Discussion

Ultimately, my initial plan of figuring out how to reduce wait times in a counseling center like the one at Appalachian State University using queuing theory proved to be a problem that would require extensive research. Instead, creating a simple simulation using data provided by the director of the counseling center at Appalachian State University allowed me to model the relationship among the number of student referrals, the number of available counselors, and the associated wait times. While not inputting as much information as would be required for a queuing model, until one is developed that can model wait times that depend on a process with repeated appointments, a simulation is more effective at representing wait times without increasing the complexity of the problem. Additionally, using a simulation allowed me to make suggestions by testing different scenarios, as was done to help Shahid Faghihi Hospital's emergency department reduce wait times [3].

As such, following the spirit of Occam's Razor, the IRS is an easily digestible model of a counseling center, yet detailed enough to make observations. In this case, although the original IRS does not leave students waiting for over four weeks, there are still students who are not attended to despite receiving a referral. If increasing staff is not possible, the Bottom Heavy IRS would allow all students who do not leave the simulation to be attended to, and it would reduce wait times. If hiring an additional part-time counselor is feasible, simply having an additional ten appointments available each week allows every student to be seen in the simulation, with no loss of students during the break. It also reduces the wait times in comparison to all the

Week	Two weeks waiting	Four weeks waiting	Six weeks waiting
0	0.00	0.00	0.00
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	0.00	0.00	0.00
7	0.00	0.00	0.00
9	0.00	0.00	0.00
11	0.00	0.00	0.00
13	0.00	0.00	0.00
15	6.06	0.00	0.00
Break	Break	Break	Break
1	0.00	0.00	0.00
3	0.00	0.00	0.00
5	0.40	0.00	0.00
7	40.18	0.00	0.00
9	53.38	0.00	0.00
11	62.16	0.00	0.00
13	46.08	12.41	0.00
15	33.92	0.00	0.00

Figure 11: Part-Time IRS Waitlist

??

other simulations. However, considering 88% of college counseling centers have reported experiencing staff shortages, I assume that if hiring more staff were possible for most counseling centers, that would be their first solution [7]. Whichever suggestion a counseling center run similarly to the simulation were to take, reducing the wait times and attending to more students would have a positive impact on student life, not only in the realm of mental health, but also would support students academically [1] [4].

8 Future Directions

Director Hogan mentioned that Appalachian sometimes utilizes graduate students who need field experience in counseling. If it were possible, rotating graduate students who can increase the amount of appointment per week, even if just ten appointments more per week, would allow more students to be seen and help prevent a large wait list, as simulated by the Part-Time IRS. However, having unlicensed staff such as graduate students hold appointments could actually work against the goal. Unlicensed staff typically have to be trained and supervised which increases the load on licensed staff. The time licensed staff would have to take to train and supervised unlicensed staff would take away from the number of appointments they could hold per week. If the benefit of adding temporary or unlicensed staff outweighs the cost of training and supervising, then utilizing graduate students would be the best option. Otherwise, shifting staff to the semester or weeks when the wait time builds up the most could be the best method to reduce the overall wait time, as simulated by the Bottom Heavy IRS.

Although a simulation was the best way for me to model the wait at college counseling centers at this time, developing a queuing model that applies to settings that have to factor in repeated appointments would help more accurately identify where wait times build up and locate when they are over- and under-staffed. Currently established queuing models are capable of representing settings with multiple phases before a person leaves the model with established queues between each phase that are mostly independent of one another, so the original queue is

only affected by the number of people in the first phase. Queuing models with multiple phases are unable to model a counseling center since although the repeated appointments could be represented as different phases, the original queue wait times are affected by the number of people attending appointments at a particular time. However, with research, a queuing model with multiple phases and a queue that is dependent on the number of people within each phase could potentially be developed. Once such a queuing model is developed, counseling centers and other similar settings could be represented and analyzed using queuing theory.

References

[1] D. Kivlighan, B. Schreier, C. Gates, J. Hong, J. Cockery, C. Anderson, and P. Keeton, *The Role of Mental Health Counseling in College Students' Academic Success: An Interrupted Time Series Analysis*, Journal of Counseling Psychology (2020), 1–9.

[2] F. Hillier and G. Lieberman, *Introduction to Operations Research*, 10th ed., McGraw Holl, Inc., 2015.

[3] H. Haghighinejad, E. Kharazmi, N. Hatam, S. Yousefi, S. Hesami, M. Danaei, and M. Askarian, *Using Queuing Theory and Simulation Modelling to Reduce Waiting Times in An Iranian Emergency Department*, International Journal of Community Based Nursing and Midwifery **4** (2016), no. 1, 11–26.

[4] M. Wood, *The State of Mental Health on College Campuses*, The Journal of the Virginia Community Colleges **17** (2012), no. 1, 5–15.

[5] National Alliance on Mental Illness, *Mental Health By the Numbers*, `https://www.nami.org/mhstats`. Accessed January 19, 2021.

[6] N. Chowdhury, L. Riddles, and R. Mackenzie, *Using Queuing Theory to Reduce Wait, Stay in Emergency Department*, `https://www.physicianleaders.org/news/queuing-theory-reducing-wait-stay`. Accessed January 15, 2021.

[7] S. Gibbons, T. Trette-McLean, A. Crandall, J. Bingham, C. Garn, and J. Cox, *Undergraduate Students Survey their Peers on Mental Health: Perspectives and Strategies for Improving College Counseling Center Outreach*, Journal of American College Health **67** (2019), no. 6, 580–592.